Prediction of Iron Losses Using Jiles-Atherton Model with Interpolated Parameters under the Conditions of Frequency and Compressive Stress

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The operating conditions such as frequency of excitation, stress and temperature in electrical machines severely affect the magnetic behavior of ferromagnetic cores which translates into increase in iron losses. Physics-based hysteresis models such as the Jiles-Atherton (JA) model can incorporate the effects of operating conditions on iron losses. In addition to this, these models can be embedded in finite element simulations. In this work, we have implemented JA model to predict iron losses and the effect of frequency of excitation waveform and compressive stress on JA model parameters has been investigated. A simple approach is proposed to predict iron losses for any value of frequency and compressive stress using original JA model equation. This approach not only reduces the computational complexity of the problem but also reduces the amount of material information required.

***Index Terms*—Electrical machines, ferromagnetic materials, Jiles Atherton model, iron losses.**

I. Introduction

Ferromagnetic materials in modern electrical machines are subjected to compressive stress during manufacturing of machine cores, and high frequency excitations and high temperatures during operation. These factors contribute to iron losses and should be considered in computer-aided machine design. Physics-based models such as Jiles-Atherton (JA) model [1] are the promising candidates to incorporate the above mentioned factors. In this work, we have investigated the effect of frequency and compressive stress on the JA model parameters and this investigation leads to the reduction of computational complexity and the required material information. This work can be useful for predicting iron losses of ferromagnetic materials subjected to any value of frequency and static stress.

II. The Jiles-Atherton Model

The JA model is one of the most popular physics-based iron loss models and it explains the hysteresis loss mechanism with the help of domain wall motion. The two modes of domain wall transitions (both its bending and translational motions) result in a reversible and an irreversible component of magnetization, respectively. The total magnetization inside a material is computed using a differential equation (Eq. (1)).

$$\frac{dM}{dH} = \frac{c}{(1+c)}\frac{dM_{an}(M_s,a)}{dH} + \frac{1}{(1+c)}\frac{(M_{an}(M_s,a)-M)}{\frac{\delta k}{\mu_o} - \alpha\,(M_{an}(M_s,a)-M)}. \quad (1)$$

Where, $M_{an}(M_S, a)$ is the anhysteretic magnetization that is computed using Langevin's polynomial [1], M_s, α, a, k and c are the JA model parameters. The details of these parameters and the differential equation are given in detail in [1]. Eq. (1) is an iterative equation and is simple to implement. It offers computational and memory complexity of O(1) [2]. The model described by Eq. (1) is a frequency independent model. A dynamic version of this model was proposed in [3] by adding the eddy current and excess loss terms [4] to Eq. (1) to incorporate the frequency dependency. The resulting dynamic JA model is given in Eq. (2). It can be seen that Eq. (2) is a nonlinear function of dM/dH thus, is expensive to solve.

$$\left[k\delta - \alpha\left(M_{an}(M_s,a) - M + k\delta c\frac{dM_{an}(M_s,a)}{dH}\right)\right]\frac{dM}{dH} - \left(M_{an}(M_s,a) - M + k\delta c\frac{dM_{an}(M_s,a)}{dH}\right) + \left(\frac{\mu_o d^2}{2\rho\beta}\frac{dH}{dt}\right)\left(\frac{dM}{dH}\right)^2 + \left(\frac{\mu_o G d w H_o}{\rho}\right)^{\frac{1}{2}}\left(\frac{dH}{dt}\right)^{\frac{1}{2}}\left(\frac{dM}{dH}\right)^{\frac{3}{2}} = 0. \quad (2)$$

Jiles proposed a stress dependent model [5] by introducing an additional term H_σ in the effective magnetic field equation (Eq. (3)) [6]. H_σ is a function of the stress σ and the change in magnetostriction with respect to magnetization.

$$H_{eff} = H + \alpha M + H_\sigma = H + \alpha M + \frac{3\sigma}{2\mu_o}\frac{\partial\lambda}{\partial M}. \quad (3)$$

Where, H is the applied field intensity and α is the inter-domain coupling coefficient [1]. It should be noted here that material magnetostriction data is not readily available. Based on Eq. (3), stress dependence was included in Eq. (1) and the resulting JA model is given by Eq. (5).

$$\frac{dM}{dH} = \frac{1}{(1+c)}\frac{(M_{an}(M_s,a)-M)}{\frac{\delta k}{\mu_o} - \left(\alpha + \frac{3\sigma}{2\mu_o}\frac{\partial^2\lambda}{\partial M^2}\right)(M_{an}(M_s,a)-M)} + \frac{c}{(1+c)}\frac{dM_{an}(M_s,a)}{dH}. \quad (4)$$

In [7], the input parameters of the original JA model were considered as the function of tensile stress [7] to compute *B-H* loops of Nickel. The parameters were also considered as function of temperature in [8] to model temperature effects in hysteresis but few more material parameters need to be identified to accomplish it.

III. Method and Results

A simple approach is presented here to predict the magnetic behavior of the ferromagnetic behavior. A total of twelve *B-H* loops (B_{max} = 1.5 T) were measured for 35WW300 non-oriented steel at different values of frequency and compressive stress using Brockhaus stress based Single Sheet Tester (SST). The JA model input parameters were identified for each of measured loops using nonlinear least square method [9].

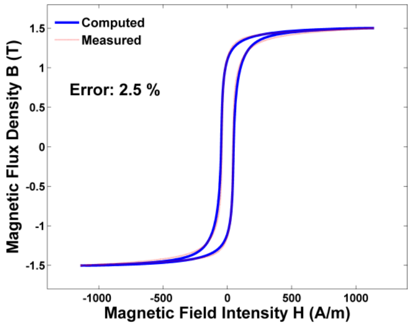

Fig. 1. Measured and computed hysteresis loop at B = 1.5 T, f = 50 Hz, and stress = 0 MPa. The identified JA parameters' values are M_s = 1.229 x 10^6A/m, a = 33.7 A/m, α = 8.77 x 10^{-5}, k = 57.9 A/m, c = 0.05. Error is defined as (ELD_{comp} - ELD_{meas})/ ELD_{meas} x 100 %. *ELD* is the energy loss density.

One of the measured loops along with the computed loop using JA model is shown in Fig. 1. An error metric is defined to check on the accuracy of the prediction. The dependence of five JA parameters on frequency and compressive stress has shown by plotting the parameter values versus frequency and stress, as shown in Fig. 2(a)-(e). Once we have this dataset available, we can use two dimensional interpolation schemes to determine the JA model parameters' values for frequency and stress. Then, Eq. (1) can be solved using these interpolated parameters to predict iron loss. One such prediction is shown in Fig. 2 (f).

The approach offers a couple of advantages. It is simple to implement, requires less material information and retains the computational complexity of the original JA model.

The same approach can be extended to take temperature effects on iron losses into account.

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IV. References

[1] D. C. Jiles and D. L. Atherton, "Theory of ferromagnetic hysteresis," *Journal of Magnetism and Magnetic Materials*, vol. 61, no. 12, pp. 48 – 60, 1986.

[2] S. Rosenbaum, M. Ruderman, T. Ströhla, and T. Bertram, "Use of Jiles–Atherton and Preisach hysteresis models for inverse feed-forward control," *Magnetics, IEEE Transactions on,* vol. 46, no. 12, pp. 3984-3989, 2010.

[3] D.C. Jiles, "Modeling the effects of eddy current losses on frequency dependent hysteresis in electrically conducting media," *Magnetics, IEEE Transactions on,* vol. 30, no. 6, pp. 4326-4328, 1994.

[4] G. Bertotti, *Hysteresis in magnetism: for physicists, materials scientists, and engineers*. Academic Press, San Diego, USA, 1998.

[5] D. C Jiles, "Theory of magnetomechanical effect," *Journal of Physics D: Applied Physics*, vol. 28, no. 8, pp. 1537-1546, 1995.

[6] H. W. L. Naus, "Ferromagnetic hysteresis and the effective field," *Magnetics, IEEE Transactions on,* vol. 38, no. 5, pp. 3417-3419, 2002.

[7] T. Suzuki, H. Baba and E. Matsumoto, "Stress effect on hysteretic magnetization curve of nickel," *International Journal of Applied Electromagnetics and Mechanics*, vol. 13, no. 1-4, pp. 307-310, 2002.

[8] A. Raghunathan, Y. Melikhov, J. E. Snyder, and D. C. Jiles, "Theoretical model of temperature dependence of hysteresis based on mean field theory," *Magnetics, IEEE Transactions on,* vol. 46, no. 6, pp. 1507-1510, 2010.

[9] P. Kis and A. Iványi, "Parameter identification of Jiles–Atherton model with nonlinear least-square method," *Physica B: Condensed Matter*, vol. 343, no. 1–4, pp. 59–64, 2004.

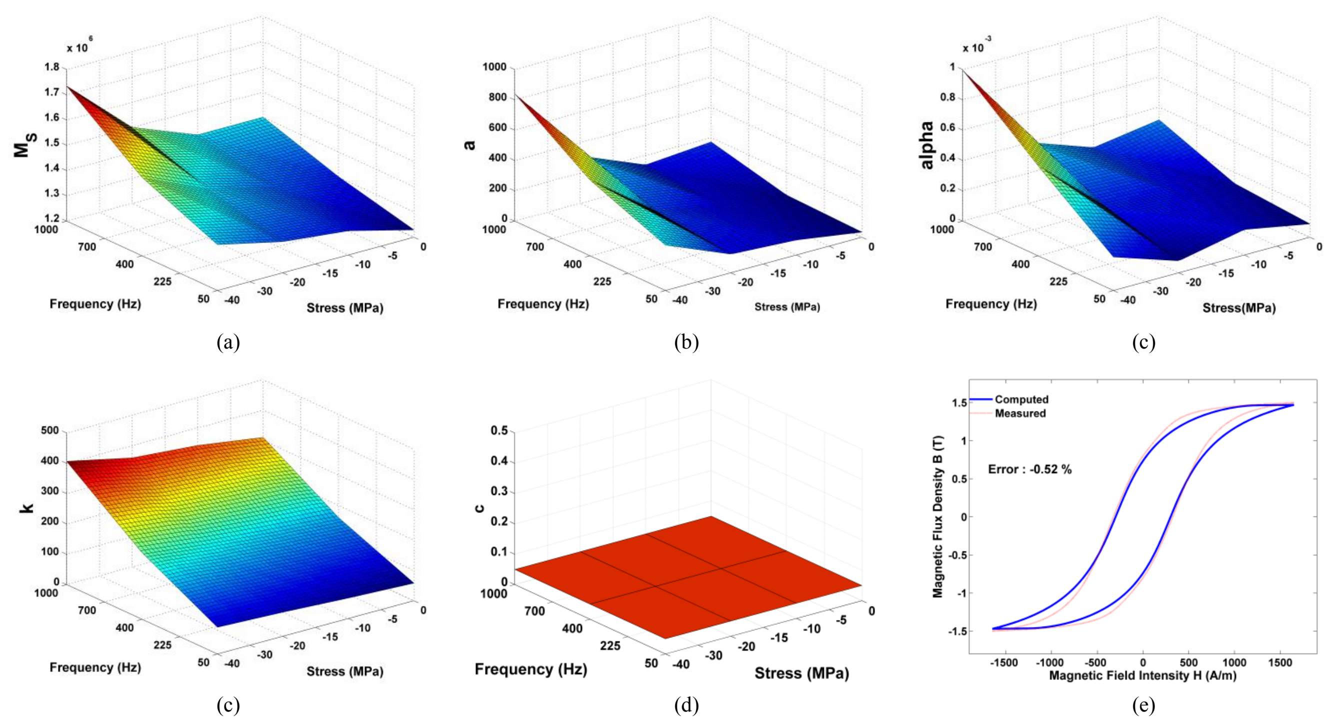

Fig. 2. The identified JA parameters are shown as a function of frequency and stress, (a) M_s (b) a (c) α (d) k (e) c. (f) Measured and computed hysteresis loops using interpolated model parameters at B = 1.5 T, f = 800 Hz, and stress = -30 MPa. The interpolated JA parameters' values are M_s = 1.583x 10^6A/m, a = 532.6 A/m, α = 6.38 x 10^{-4}, k = 327.89 A/m, c = 0.05. Hysteresis loops used as input dataset were measured at frequencies 50 Hz, 400 Hz and 1000Hz for all values of compressive stress (i.e. 0 MPa, -10 MPa, -20 MPa, and -40 MPa).